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Design concept for bolted Glass

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For the design of load carrying glazing structures the connection technique plays an important role, as e.g. there are significant stress concentrations in the vicinity of holes that are subject to a point-like support of glass panes. If no further constructional means are provided, the stress peaks cannot be redistributed and thus a sudden brittle failure is likely to occur. In particular this concerns bolts in bearings of drilled glass holes as long as no ductile stress distributing interlayers in the clearance between hole bearing and bolt shank is provided. In the article, a simple design formula for glass joints with bolts in bearings is suggested that is based on an analytical approach where the local stress distributions coming on the one hand from the bearing pressure and on the other hand from the net section stress concentrations are superposed, and both for which the solutions according to AIRY's differential equation are found resp. used further on. By this the relevant stresses in dependence on the acting design force, the hole diameter and the pane thickness can be determined without performing complex and time consuming Finite Element calculations.

Keywords: Drilled glass, Bolts in bearing, Verification, Design equation

1. Introduction

Glass is used nowadays more and more as a structural member of buildings. Nevertheless design rules are often absent, time-consuming and often expensive approval procedures will be necessary. Glass significantly distinguishes from other building materials as sudden breakage of glass panes governs the ultimate limit state due to its brittleness. Therefore the derivation of simple engineering models is complex. To create a quasi-ductility of the structure, interlayers e.g. out of mortar or synthetic material, ought to be provided allowing for the dissipation of stress peaks.

Further, special care is demanded with respect to the design of joint constructions of glass elements due to its realization with point-fixings leading to high stress concentrations in the vicinity of the holes.

This article deals with joints, where the forces act parallel to the glass plane with bolts in bearing. A consistent approach for a design model for joints with bolts in bearing is presented, that bases on the analytical calculation of stress states of complex joint geometries. Also imperfections are included that result of production and erection as well as of the conceptual design. Examples for typical detailing of those bolt fixings are shown in figure 1, where mortar is used as convenient interlayer between steel bolt and glass hole.

Figure 1a, b and c: Mortar-Interlayer to create ductility behaviour and example of a joint (CCI Munich)

2. Stress equations for bolted joints

To derive stress equations for arbitrary geometries for bolted glass joints the basics of the statics of plane load-bearing structures are needed. The three static requirements to fulfil equilibrium, compatibility and material law can be reduced to the differential equation of AIRY:

$$\Delta\Delta F = 0\,. \tag{1}$$

The resulting stress state can be spit into two basic stress states which can be linearly superposed. They serve to calculate easily any general resulting stress state of a drilled pane under arbitrary bolt loading, cf. figure 2 and figure 3. These two basic stress states will shortly be presented in the next two chapters of this article, a detailed derivation and description can be found in [1], [3] and [7].

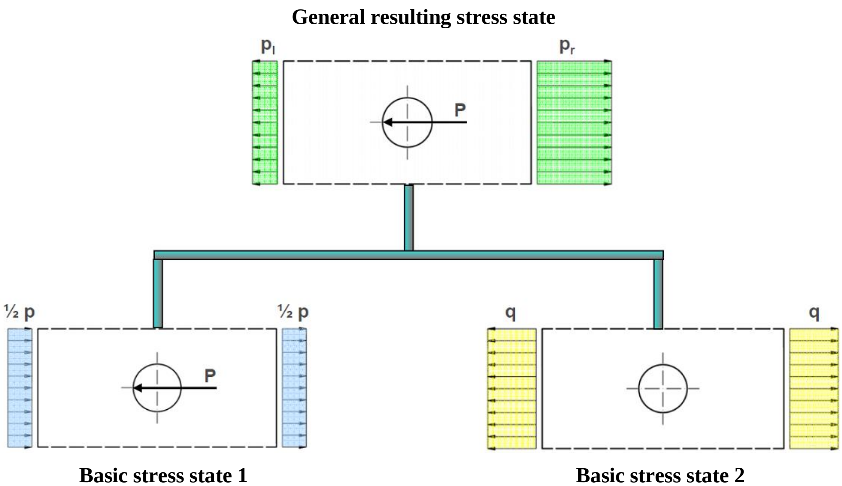

Figure 2: Linear combination of the two basic stress states
for calculation of the global resulting stress distribution.

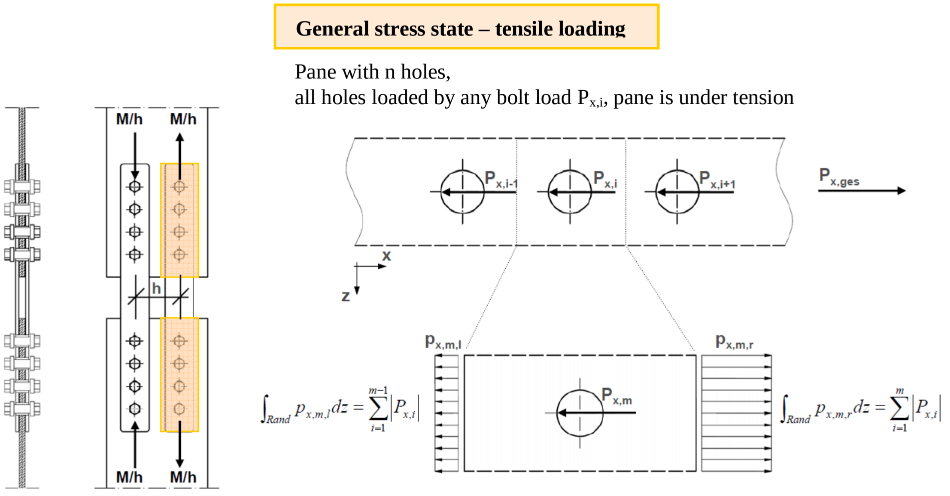

Figure 3: Joint loaded by a moment, free of shear force.

3. Basic stress state 1: Drilled pane with bolt load

For drilled panes with bolt loads an analytical solution is existent, cf. figure 4. The pressure in the vicinity of the hole (figure 4a) can be described with a cosine series as follows [4], [6]:

$$p(\varphi) = p_0 + \sum_{n=1}^{\infty} p_n \cdot \cos n\varphi \tag{2}$$

The members p_i of equation (2) are determined with Finite Element calculations. It comes out, that the influence of the pane width on the stress can be neglected until a width of $B \geq 3d_o$, where d_o is the hole diameter.

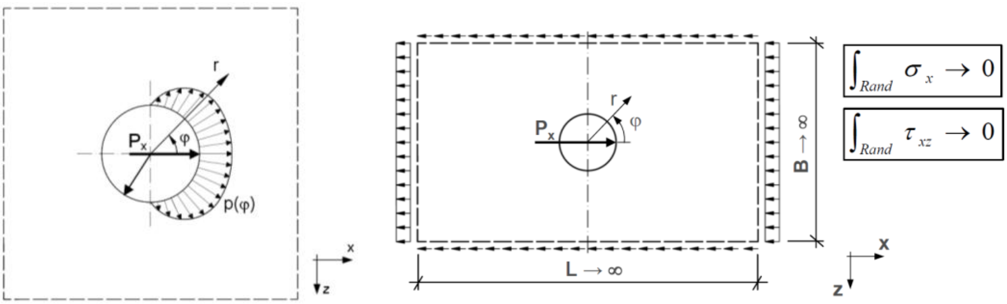

Figure 4a and b: Stress distribution caused by bolt load and equilibrium of a drilled pane, loaded by bolt pressure.

Thus the following equation (3) to calculate the decisive tangential stress is an allowable solution for a pane with a limited dimension under bolt load, whereby the load is respectively transmitted by both fixed boundaries, see figure 5. The further stress equations for radial stress and shear stress can be taken from [1] and [7].

Tangential stress

$$\sigma_{\varphi,basic1}(r,\varphi) = +\frac{p_0}{t}\cdot\frac{a^2}{r^2}+\frac{1-\mu}{4t}\,p_1\,\frac{a}{r}\left(1+\frac{a^2}{r^2}\right)\cos\varphi + \\ +\frac{1}{2t}\sum_{n=2}^{\infty} p_n\,\frac{a^n}{r^n}\left((n-2)-n\frac{a^2}{r^2}\right)\cos n\varphi \qquad (3)$$

where:

r, φ: Polar coordinate system based on the centre of the hole
p_i: Term of the series expansion of the bearing load
a: Radius of the hole
t: Pane thickness
μ: Poisson's ratio of the pane

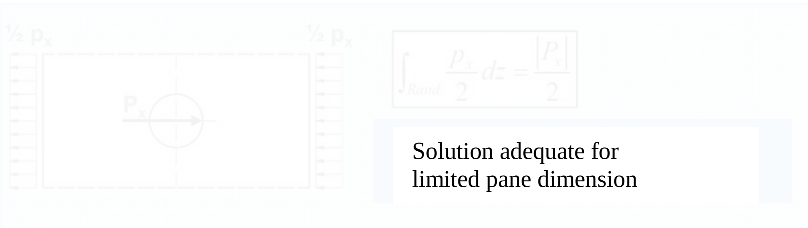

Figure 5: Basic stress state 1.

4. Basic stress state 2: Drilled pane tensile-loaded without bolt load

Basic stress state 2 consists of a drilled pane, symmetrically tensile-loaded by line loads ½ p_x, where the resulting boundary loads are half of the bolt load of the basic stress state 1, cf. figure 6.

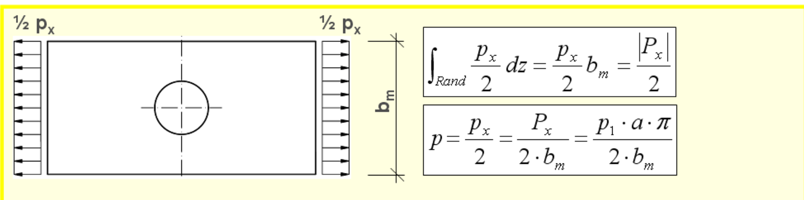

Figure 6: Basic stress state 2.

Unlike to the basic stress state 1, here, the pane width is essential and the influence on the stresses in the glass must not be neglected. Hence a new parameter, the relevant pane width b_m depending on the shortest edge and hole distances (according to design rules of steel constructions) is introduced to make the solution possible also for complex joint geometries, see figure 7.

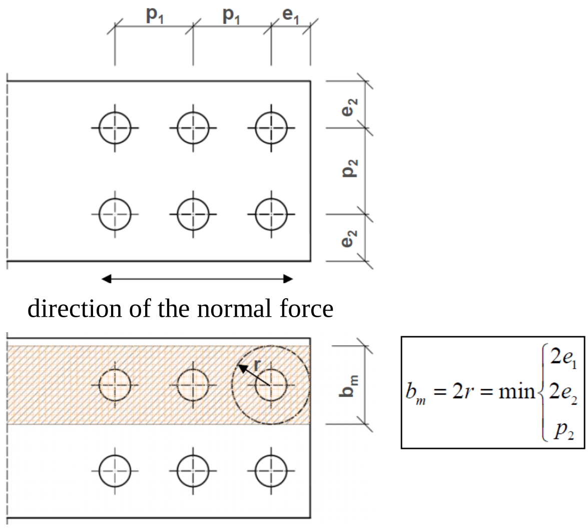

Figure 7: Definition of the relevant pane width b_m.

The tangential stress can be calculated with the following equation (4), the equations for the two further components (radial and shear stress) can be seen in [1] and [7].

$$\sigma_{\varphi,basic2}(r,\varphi) = \frac{p_1 \cdot a \cdot \pi}{4t \cdot b_m} \cdot \left[1 + \frac{a^2}{r^2} - \left(1 + \frac{3a^4}{r^4}\right) \cdot \cos 2\varphi\right] \tag{4}$$

Where:

r, φ: Polar coordinate system based on the centre of the hole
p_1: First term of the series expansion of the bearing load
a: Radius of the hole
t: Pane thickness
μ: Poisson's ratio of the pane
b_m: Relevant pane width

5. Total stress state

To calculate now the total stress state, basic stress state 1 and basic stress state 2 have to be linearly combined as mentioned before and shown in figure 8. Thus arbitrary joint geometries and loading scenarios can be evaluated using equation n^o (5) for the tangential stress. Also here, reference should be taken to [1] and [7] for the equations of the two further total stress components (radial and shear stress).

$$\sigma_{\varphi,total}(r,\varphi,\xi) = \begin{aligned}[t] & +\frac{p_0}{t}\cdot\frac{a^2}{r^2}-\frac{1-\mu}{4t}\cdot p_1\cdot\frac{a}{r}\cdot\left(1+\frac{a^2}{r^2}\right)\cdot\cos\varphi+ \\ & +\frac{1}{2t}\cdot\sum_{n=2}^{\infty}p_n\cdot\frac{a^n}{r^n}\left((n-2)-n\cdot\frac{a^2}{r^2}\right)\cdot\cos n\varphi+ \\ & +K_m\cdot\frac{p_1\cdot a\cdot\pi}{4t\cdot b_m}\cdot\left[1+\frac{a^2}{r^2}-\left(1+\frac{3a^4}{r^4}\right)\cdot\cos 2\xi\right] \end{aligned} \qquad (5)$$

where:

r, φ:	Polar coordinate system based on the centre of the hole
p_i:	Terms of the series expansion of the bearing load
a:	Radius of the hole
t:	Pane thickness
μ:	Poisson’s ratio of the pane
b_m:	Relevant pane width
K_m:	Multiplication parameter of basic stress state 2

The parameter K_m depends on the static system of the joint and can be calculated as shown in figure 8 and figure 9. As simplification the joints are reduced into strips.
A new running coordinate ξ is introduced, because only the forces in x-direction are considered calculating stress state 2.
The analytical solution, equation (5), has been compared to Finite Element calculations and it was shown, that the results coincide with good accuracy, cf. figure 10, so that the presented formula can be seen as an adequate solution for engineer practice.
Furthermore the results could be verified with numerous experimental tests on glass panes under normal load as well as under combined loading of shear and bending.

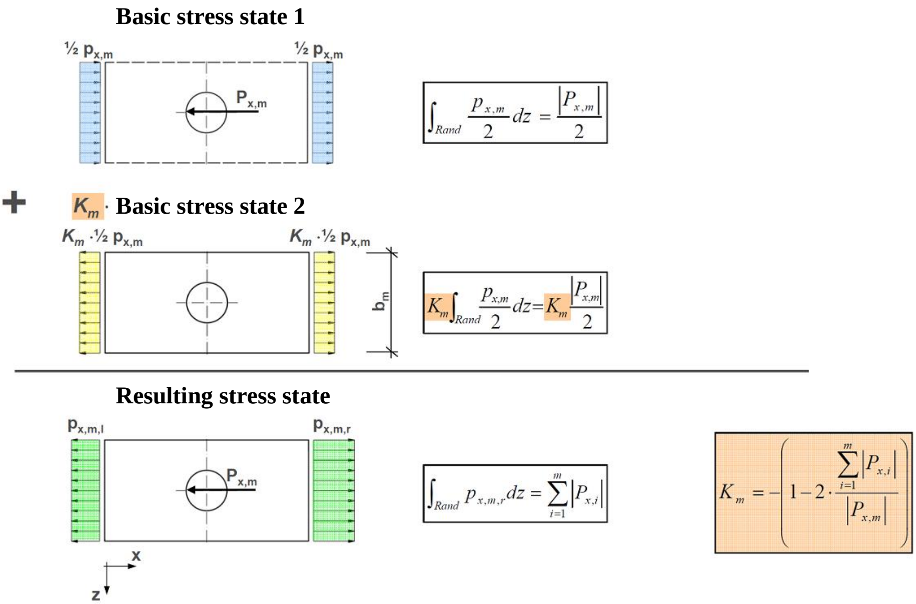

Figure 8: Derivation of the resulting stress state and determination of the parameter K_m for joints loaded by normal load.

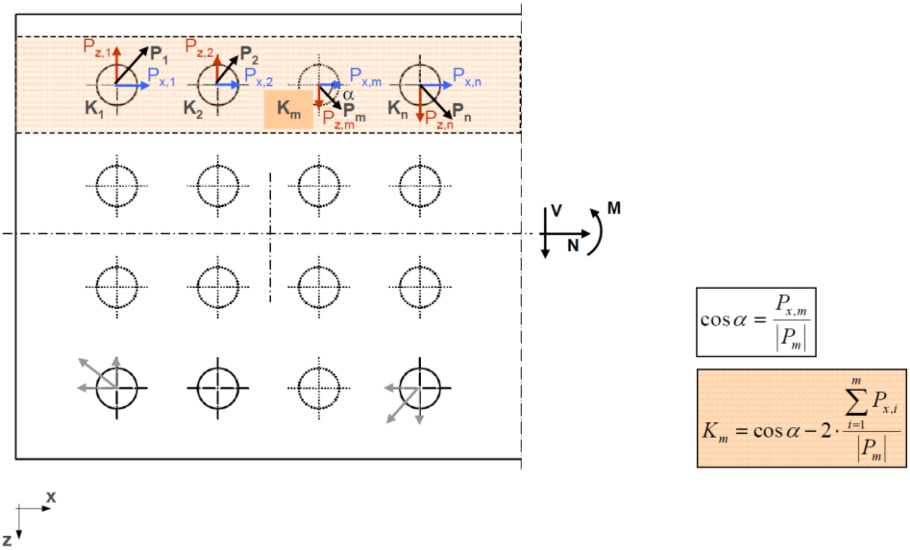

Figure 9: Determination of the parameter K_m for joints loaded by normal and shear load as well as by moment at example of investigated strip.

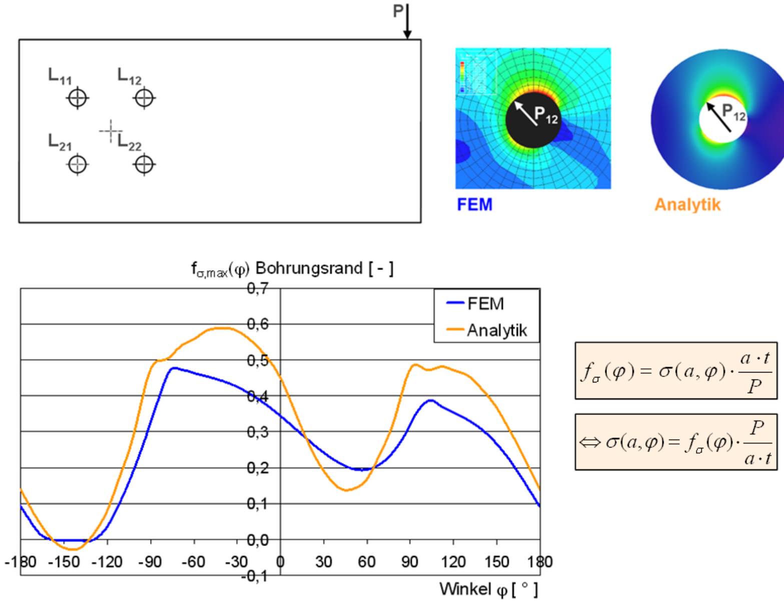

Figure 10: Comparison of results of analytical to Finite Element calculations for a joint in bending and shear, evaluation of hole L_{12}.

6. Simplified design formula

Finally the analytical method could be transferred into a simple design equation (6), as the two basic stress states 1 and 2 could be reduced to the quotient $P/(d_o \cdot t)$. With this equation now the decisive stresses in the edge of the hole can be calculated in dependence of the design bolt load, the hole diameter and the glass thickness. The influences of design parameters of the joint as well as production and erection tolerances on the maximal stress are considered in terms of coefficients k_i, which have been systematically determined through extensive Finite Element calculations [1].

$$\sigma_{\varphi,\max,d} = \prod_{i=1}^{5} k_i \cdot \left(1{,}2 + 2{,}2 \frac{K_m}{b_m} \right) \cdot \frac{P_d}{d_o \cdot t} \leq \sigma_{zul} \qquad (6)$$

where:

P_d:	Resulting design load at the decisive hole
d_o:	Hole diameter
t:	Glass thickness (of one glass pane if used laminated glass)
k_i:	Coefficients to consider the design of the joint as well as production and erection tolerances
b_m:	Decisive pane width in [d_o]
K_m:	Multiplication parameter of basic stress state 2
σ_{zul}:	Design resistance of the glass element

Design formula (6) now should be used as follows:

a) Determination of the internal normal and shear forces as well as moment
b) Distribution of the internal forces and moment on the single bolts depending on the polar moment of inertia and the non-uniform distribution of the normal force over the joint length
c) Determination of the relevant pane width b_m
d) Compilation of the coefficients k_i
e) Calculation of the parameter K_m

The design model bases on the concept of partial safety factors as foreseen in the future German design code DIN 18008 “Glas im Bauwesen - Bemessungs- und Konstruktionsregeln” [8]. For completion of the design proposal the partial safety factors γ_M for the material resistance of toughened and heat-strengthened glass have to be determined. For the presented design formula γ_M has been derived according to EN 1990 Annex D. In this code a method is presented to compare the load carrying capacity calculated by the new design model r_t with the results of the experimental investigation r_e. The diagrams shown in figure 11 present the inverse of the cumulative frequency of the Gaussian distribution over the quotient r_e/r_t for toughened glass and heat-strengthened glass.
γ_M can now be determined and with the aim to obtain consistent values for the both glass products γ_M = 1,3 is proposed. The derivation of the partial safety factor bases on the glass strength 80 MPa for toughened glass and 70 Mpa for heat-strengthened glass [5].

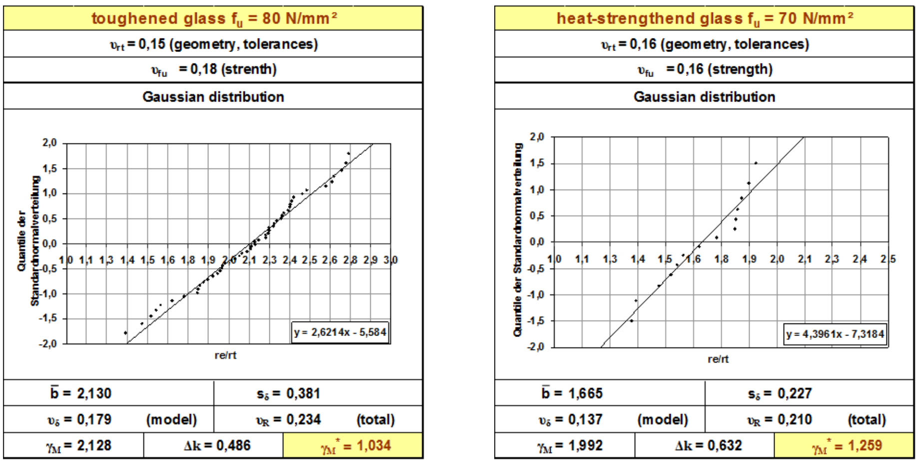

Figure 11: Derivation of the partial safety factor γ_M for toughened glass and heat strengthened glass according to DIN EN 1990 Annex D with 75% confidence probability.

7. Summary

The design of bolts in bearings of drilled glass panes is now significantly simplified due to the results of recent research projects [1], [7]. Stress states of a drilled glass pane loaded by bolt loads can now be calculated with a simple design equation without the use of complex Finite Element calculations.

Further, the design concept could be verified by numerous experimental investigations, especially for systems loaded by normal force. So far, several tests have been fulfilled also for more complex systems loaded by bending moment and the solution could be approved as a method with conservative results.

8. References

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